Service provision and loans: Price and risk implications
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Service provision and loans: 
Price and risk implications

Production de services bancaires et crédit bancaire: 
Implications en termes de prix et de risque

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**Abstract:** Deregulation of the banking system has increased competition and prompted wide changes in the activities of banks. As revenue from intermediation activities of banks has decreased, banks have broadened the range of products they offer to their clients, which generate revenue other than interest margin. This paper offers a complementary explanation of the link between intermediation activities and service provision. We show that banks may be willing to decrease their lending rate, using loans as loss leader, and take on higher credit risk, in order to capture clients to whom they can sell services.

**Résumé :** La déréglementation du système bancaire a eu pour conséquence une augmentation de la concurrence entre banques. Les banques, face une diminution de leur revenu d’intérêt, ont cherché d’autres sources de revenu, en modifiant notamment l’offre de services à leurs clients. Ce papier a pour objectif de compléter l’analyse du lien entre activités d’intermédiation et production de services bancaires. Nous montrons que les banques, pour attirer des clients à qui elles vendront des services, sont susceptibles de diminuer le taux débiteur, utilisant les prêts comme produit d’appel, prenant ainsi plus de risque.

**JEL Classification:** G21, G10

**Keywords:** bank interest margins, fee-based activities, asymmetric information
1. Introduction

As a result of deregulation, the banking industry has experienced major changes over the last three decades in the form of increased competition, concentration and restructuring. This prompted among others increasing competition both within the industry and from financial markets, and thus an increase of the funding costs of banks as shown by Schmidt et al. (1999). As a consequence, bank interest margins sharply declined, implying a change in the structure of banks’ income statement, and, in particular, a decrease in the share of interest revenue (ECB, 2000, Edwards and Mishkin, 1995, Lewis and Pescetto, 1996, Plihon, 1998, Stiroh, 2004). However, banks have reacted to the new environment by adopting a proactive strategy (ECB, 2000), seeking for new sources of revenue. Banks have widened the range of products they offer to their clients, sometimes entering new markets. The development of services, we consider in this paper transaction services supplied by retailed banking to small and medium–sized firm (checking, cash management, bank account management, data processing, letters of credit, …), has been therefore undertaken by banks as an alternative source of revenue, to offset the decrease of traditional intermediation revenue. The present paper theoretically investigates the effect of diversification on the price and quality of the loan portfolio of banks. Specifically, we analyse how the sale of services impact on loan pricing and banks’ risk taking.

Given the evolution observed in banks’ income structure, two questions naturally arise: (1) How has the role of banks been modified? (2) How has the bank-customer relationship been affected? Our view is that recent years may have seen the development of a new strategy: banks use loans as loss-leader as they anticipate the potential sale of services in future periods to core clients. An issue similar to ours has
been addressed by Cukierman (1978), who shows that banks ration the supply of credit to borrowers who buy services less. This paper is also related to the current American and European literature on tying and cross-subsidisation (Drucker and Puri, 2002, GAO, 2003, Lepetit et al.1, 2007) which empirically underline such effects. In this paper, we aim to model the effect of the sale of services on lending, using a principal-agent structure. More precisely, we focus on banks/borrowers’ incentives to control the riskiness of projects funded by banks and more generally their implications on banks’ risk taking. We find, on the one hand, that banks may decrease the lending rate they offer, using loans as loss-leader, and on the other hand, that banks may have fewer incentives to screen borrowers to assess their riskiness, increasing their risk taking. When taking into account commission and fee-based activities, studying banks’ behaviour changes appears essential.

The remainder of the paper is organised as follows. Section 2 briefly reviews the literature on banks considered as multi-product firms. We present, in section 3, a model where the supply of loans and service provision are considered as banking activities, which is resolved in section 4. Concluding remarks are presented in section 5.

2. Related literature

Multi-production and cross-selling have been studied in the banking literature, starting with the literature on implicit interest. One of the difficulties concerning the multi-product provision of banking services is that the pricing of services has been subject to government interference/regulation. For example Klein (1971), Barro and Santomero (1972) and Tarkka (1995) have studied the demand for deposits and have pointed out that when the government imposes an interest rate ceiling on current and

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1 In this study on European banks, authors empirically explore the link between product expansion and the pricing of loans.
saving accounts, banks pay an implicit interest rate by setting charges for services below the competitive price.

Joint production in banking can also be achieved through the production of deposits and credits. Chiappori et al. (1995) underline the emergence of subsidies between regulated deposit rates and lending interest rates. They also argue that without a deposit interest rate ceiling, the credit rate increases. Melnik and Plaut (1986) discussed the existence of the bundle of loan terms, but which are made under loan commitment contracts, which enable borrowers to trade-off more favourable values of some loan variable for less favourable value of some other loan variable. In the literature, the relationships between service provision and deposit interest rates, between deposit and credit rates or between loan terms, have been widely considered, in contrast to the relationship between service provision and lending rates.

Regulation which sets deposit interest rates ceiling, implies lower service charges and/or lower credit interest rates than the ones that prevail in the absence of regulation. Nevertheless, if cross-subsidisation is stressed in these models, this one arises in the aftermath of regulation. However as a consequence of deregulation, banks have increased the share of the cost of service provision they charge to clients (Jacolin and Pasquier, 1995, and DeYoung and Roland, 2001). From implicit interest rate payments, banks have tended to switch to the payment of explicit interest rates and the billing of services provided.

Not so much attention has been given to services and lending rates. The production of banking services and credit has been theoretically addressed by Cukierman (1978).

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2 Under regulated deposit rates, banks will decrease their lending rate if they can sell tied-up contracts, which stipulate that agents applying for a loan must simultaneously deposit their cash balances in the bank.

3 Another issue could be the effect in terms of efficiency. When banks charge the real price for services they provide to their clients, they eliminate cross-subsidisation between clients.
He shows that customers who have the highest marginal propensity to buy banking services, when granted credit, will not be credit rationed. Indeed as the borrowers’ demand for services is an increasing function of the amount of loans they receive, banks will choose to grant credit to those who have a high propensity to buy services. Cukierman also shows a decrease in the lending rate when such a demand for services is taken into account. However, in his work, no attention has been paid to loan default under asymmetric information between lenders and borrowers. Recently, even though not theoretical, the contribution of Drucker and Puri (2002) underlines a possible determinant of cross-subsidisation between services and the lending rate: they examine the practice of tying, which occurs when a bank provides a loan to an issuer, in order to secure underwriting business.

3. Theoretical model presentation

The model developed builds on the article of Covitz and Heitfield (1999). We have transposed their setting to the bank-firm relationship, allowing us to take into account service provision by banks in addition to intermediation activity. The key feature of the model developed in this section is the introduction of services in the profit function of banks. In our model we focus on the relationship between a borrower and his bank.

Consider the following sequence of events. At time T=0, the firm chooses the project it wants to undertake and therefore the risk it takes, and then applies for a loan. Then the bank decides whether or not to engage in screening. At time T=1, the borrower earns a return on his project, if it has been successful and if it has been funded. While lending to the firm, the bank develops a banking relationship with its client that may

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4 However, unlike these authors, we are not investigating here the effect of the supply of financial services (investment and/or merchant banking) on loan price.

5 The issue they were concerned with related to moral hazard between borrowers and banks, on the one hand, and between banks and the deposit insurance authorities, on the other hand.
potentially enable the bank to sell services at time $T=2$. Our aim is to understand how selling services can alter the firm’s behaviour towards project risk and the bank’s pricing and screening incentives.

3.1. Agents

3.1.1. Firm

3.1.1.1. Project

The firm is managed by an entrepreneur who may undertake two kinds of project: a low risk project which yields a gross return $l$ with a probability $\gamma$, $\Pr(\text{return} = l) = \gamma$, or zero with a probability $(1-\gamma)$, and a high risk project which yields either a gross return $h$ with probability $\theta$ or zero with probability $(1 - \theta)$, $\Pr(\text{return} = h) = \theta$.

The relationship between the distributions of the return of projects is governed by the following assumptions:

- $l < h$, the return of the low risk project is lower than the return of the high risk project,

- $\gamma > \theta$, the probability that the low risk project is successful is higher than the one of the high risk project.

3.1.1.2. Funding

To finance its project, the (non-bank) firm needs one monetary unit, which is borrowed entirely from its bank.\footnote{The firm cannot be financed by equity contract. Of course, if such a possibility was considered, then the conflict of interest between the firm and the bank would be lessened, but a new form of agency cost would arise between the entrepreneur and the new outside equity holders.} The firm will repay its loan at time $T=1$ and is assumed to be risk neutral.
The firm is subject to limited liability. We assume that the value of the entrepreneur’s equity is normalised to zero. In case of firm failure, the value of the loan is equal to the value of the remaining assets, which are kept by the bank. For simplicity, the value of the remaining assets is normalised to zero.

3.1.2. Bank

Let \( r_f \) denote the gross risk-free interest rate in the economy, and the interest rate paid on current accounts. This cost of funds also represents the opportunity cost for a bank. The bank is assumed to be neutral towards risk.

The bank operates under limited liability: if it fails, it does not have to entirely reimburse depositors. However deposits are guaranteed by a deposit insurance system, hence depositors do not monitor banks. The price of the deposit insurance is fixed and normalised to zero. Because of the existence of a deposit insurance scheme with a fixed rate premium, the bank may choose to take on too much risk, and in this case we would say that it adopts a strong moral hazard attitude towards the deposit insurance fund. We suppose that prudential regulation is in place to limit this risk.

The following condition is necessary to guarantee that both projects have a positive probability of being undertaken and that the bank will lend funds to an entrepreneur who wishes to undertake one of them:

**Assumption 1**

\[
0 < \gamma (1 - r_f) < \theta (h - r_f)
\]

where \( r_f \) stands for the risk-free interest rate.

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7 Another comparable situation could be the opening of a credit line by the bank. Then, if the bank decides to screen and if it detects a high risk project, it can close the credit line or require a refund if the option of borrowing has been exercised in the meanwhile.
This assumption implies firstly, that from the firm point of view, the high risk and low risk projects have a higher return than the risk-free asset, and secondly that the firm has an incentive to choose the high risk project.

### 3.2. Adverse selection

Given the risk neutrality of both agents, the limited liability of the bank and assumption 1, on the one hand the firm will always have the incentive to choose the high risk project, and on the other hand, the bank would not prefer the firm to choose the low risk project.\(^8\)

Given the existence of a deposit insurance scheme, there is a social cost to the bankruptcy of the bank. Let us call \(C\) the cost of the bank’s failure. We assume that the low risk project is socially valuable:

\[
\gamma l - (1 - \gamma)C > \theta h - (1 - \theta)C \iff \gamma l > \theta h - (\gamma - \theta)C
\]

(1)

The term \((\gamma - \theta)C\) is the additional cost due to bank failure when the high risk project is chosen instead of the low risk one. Therefore there is a conflict of interest between the optimal social choice of the project and the one made by the firm which is funded by the bank.

In order to prompt the bank to valuate the low risk project, the banking sector is regulated (prudential regulation), and therefore, the bank needs to hold an amount \(k\) of shareholder capital in proportion of total assets. Formally let \(k\) stand for the ratio of capital to total assets. A bank needs to collect only an amount \((1 - k)\) of deposits to lend one monetary unit to a firm.\(^9\)

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\(^8\) In such a situation, the bank knows that the probability that the firm chooses the low risk project is equal to zero. Thus the lending rate will be set given this information and depending on the lending market structure. Given its limited liability, the bank will not look for the firm to undertake the low risk project.

\(^9\) We ignore any deposit reserve requirement.
Therefore given the regulation of the banking industry, a possible conflict of interest between the bank and the firm can arise. Indeed if the firm has an incentive to always undertake the high risk project, the bank may prefer the firm to undertake the low risk project or not, depending on the value of the lending rate.

One way for the bank to induce the firm to choose the low risk project is to screen a loan it may agree upon, but it has a present discounted cost equal to \(s\). Ex-ante the bank cannot observe which project is chosen, but it can screen at time \(T=0\), i.e. before granting a loan, at cost \(s\) (the project return is realised at time \(T=1\)).

If the firm has chosen to undertake the high risk project, screening will enable the bank to detect it, and thus it will not lend funds to the firm.

If the bank observes that the high risk project has been chosen, it will not offer the firm a loan to undertake the low risk project. The reason is a credibility constraint: if the bank lends funds for the low risk project after detecting that the firm had planned to undertake a high risk project, then the strategy of the entrepreneur would be to always consider the high risk project first, and then, if screened, to move on to the low risk one.\(^{10}\) Alternatively one could assume that the initial choice of a project is irreversible. To encourage the firm to choose the low risk project, the bank must exclude it in case it chooses the high risk project, but again, this is a state that the bank can only observe by screening.\(^{11}\)

**Assumption 2**

\[
s < (\gamma - \theta) k_r < \theta h
\]

\(^{10}\) Screening and sanctions would then be necessary to assure that the firm chooses the low risk project.

\(^{11}\) To satisfy the condition that the bank will find it more profitable to implement screening, the interest rate that the bank charges for loans to the firm, \(r\), should be not too high. We will see later that this condition is satisfied when screening is a profitable option for the bank.
\((\gamma - \theta).k.r\) represents the ex-ante additional cost for the bank when the firm’s project fails, and the borrower (the firm) has undertaken the high risk project instead of the low risk one.

Given that \(s\) is the cost of screening, the first inequality of the assumption is a necessary condition for screening to occur with some positive probability. It states that the cost of screening a loan is smaller than the expected loss in capital to a bank that lends to an entrepreneur who undertakes the high risk project. Assumption 2 ensures that the bank has an actual incentive to screen the firm at cost \(s\). However the second inequality means than the expected additional loss in capital is smaller than the expected return of the high risk project. In other words, depending on the value of the lending rate, the bank may find profitable that the firm undertakes the high risk project even if the probability that this project fails is higher than the one of the low risk project.\(^\text{12}\)

Also, to rule out the possibility that it is never profitable to undertake the low risk investment, we suppose:

**Assumption 3**

\[ \% < 1 \]

The expected gross return of the low risk investment is high enough for the firm to repay the loan. This condition is sufficient but not necessary.

\(^\text{12}\) As it will become clear below, the bank may not have always the incentive to screen especially if it can capture all the return of the high risk project (cf. section 4).
3.3. Sale of services

In this model, the bank is considered to be a specialist enterprise which sells loans and services (the latter is defined as generating commissions and fees).

The bank has the possibility to sell services to its borrower. Services offered might be payment transactions, securities transactions, guarantees, current account management, credit card business, consulting activities… An entrepreneur (a firm), which has a need for services, can address his demand to different providers. However he has an incentive to address it to the bank lending to him, if we assume that the firm suffers disutility from a multiplicity of suppliers. The disutility, that the agent suffers from, can be explained by transport cost and/or switching cost. The price set for services, in such an environment, is the cost of producing services plus the agent’s disutility. In other words, the price set for services is above the marginal cost. In the case of this model, when the bank grants one unit of loan to a borrower, it can sell at most one service to this potential client. The sale of services, at $T=2$, is conditional upon the success of the entrepreneur’s project, at $T=1$. Indeed in the case of an unsuccessful project, the borrower fails and he is unable to buy services in future periods. The expected discounted level of both commission and fee income, earned from this

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13 We take the demand for services by the firm as granted. We do not aim, in this model, to make it endogenous.

14 With regard to the issue of transport cost, see Rochet (1992), Grimaud and Rochet (1994), and Saïdane (1997). One could assume, for example, a spatial competition model to formalise the service activity (transport cost can be also associated to differentiated products, from one seller to another). However the purpose of our model is not to model the price of services. Modelling the demand of services as explained before will not make the demand endogenous, which is not our aim.

15 With regard to the issue of switching cost, we refer to the survey of Klemperer (1995) for a general explanation of the phenomenon. Switching costs occur in banking, for example, because of the cost of closing an account with one bank and opening a new one elsewhere or because it may not be worth incurring the information costs to shop for a better deal when the bank client makes repeated small purchases to its bank (Kim et al., 2003, Rhoades, 2000). As for example, Stango (2002) in his study identifies switching costs as a major determinant of credit card pricing for commercial banks.
activity, is denoted by $f$. $f$ represents the gross expected discounted level of commission and fee revenue for the bank, and is defined as follows:

$$f = c + t$$  \hspace{1cm} (2)

where $c$ is the expected discounted level of cost of producing services, and $t$ the expected present level of the transport cost and/or switching cost. Thus $t$ is a measure of the disutility that the agent suffers when changing provider of services and can be interpreted as a monopoly margin. In other words, the bank will be able to make a profit on its service activity, which equals to $t$ per borrower.

The cost of services for the firm, $f$, does not appear in its profit function as we assume it is part of the cost of its production activity. In other words, the fact that the firm buys services does not affect the gross return of its project.

To summarise, we have the following sequence of events: at $T=0$, the firm chooses its project risk \{low risk, high risk\} and then applies for a loan from the bank. The bank decides or not to screen the demand for loan. If it does so, and discovers that the firm considers a high risk project, it decides not to grant the loan; at $T=1$, if the firm has been successful whilst applying for a loan, then the project return is realised; at $T=2$, the bank charges $f$ for services, if the project, and thus the loan, has been successful.

The bank sets the lending rate, depending on the credit market structure assumption, before the game takes place. When a monopoly on the lending market, the bank is price-maker, and in the competitive case the bank is price-taker. Both the entrepreneur and/or the bank can accept or refuse this credit rate. If they agree on the rate, the bank has two possibilities; (i) it screens the loan, and if the high risk project has been chosen, it decides not to grant the loan; or (ii) it does not screen the loan.
The common practice is to solve this game by backward induction.\textsuperscript{16} We should first determine the value of the price of services, $f$, but we take it as given knowing that the bank sets it above the marginal cost. Thus we first determine the probabilities that the firm chooses the low risk project and that the bank screens for a given level of the lending rate. Then, given the probabilities that the firm chooses the low risk project and the bank screens the loan, interest rates are determined as agents’ behaviour is anticipated. All the values found are part of the same equilibria.

4. Model resolution

4.1. Incentives to screen and level of project risk

In this subsection, commission and fee income, as well as the lending rate are taken as given.\textsuperscript{17} We aim to determine the equilibrium, in which the bank screens the loan with a probability $p_s$ and the firm chooses the low risk project with a probability $p_l$. To do so, we need first to characterise agents’ strategy.

4.1.1. The bank and firm’s profits

We first present the bank and firm’s profits in order to determine the behaviour of both agents. Let us envisage the bank and firm earnings, respectively, in the pay-off matrix:

\textsuperscript{16} See appendix A for a game tree.

\textsuperscript{17} We do not consider in this section the effects of the sale of services on the lending rate. This will be done in the next section.
Firm screening ($p_s$) not screening ($1 - p_s$)

<table>
<thead>
<tr>
<th>Firm</th>
<th>low risk project ($p_l$)</th>
<th>high risk project ($1 - p_l$)</th>
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 Bank profit:

- when the bank screens the loan, its expected profit ($\pi_s^B$) is:

$$\pi_s^B = p_l \gamma(r-r_f + t) - p_l (1-\gamma)k_r - s$$  \hspace{1cm} (3)

- when the bank does not screen the loan, its expected profit ($\pi_{ns}^B$) is:

$$\pi_{ns}^B = p_l \left[ \gamma(r-r_f + t) - (1-\gamma)k_r \right] + (1 - p_l) \left[ \theta(r-r_f + t) - (1-\theta)k_r \right]$$ \hspace{1cm} (4)

Firm profit:

- when the low risk investment is chosen, the expected firm’s profit ($\pi_l^F$) is:

$$\pi_l^F = p_s \gamma(l-r) + (1-p_s) \gamma(l-r) = \gamma(l-r)$$ \hspace{1cm} (5)

- when the high risk project is chosen, the expected firm’s profit ($\pi_h^F$) is:

$$\pi_h^F = p_s \times 0 + (1-p_s) \theta(h-r) = (1-p_s) \theta(h-r)$$ \hspace{1cm} (6)
If we analyse the bank and firm’s behaviours, we observe that when the firm chooses the low risk project, the bank’s profit is always higher when it has decided not to screen (because of the cost of screening). Therefore we first concentrate on the case where the firm chooses the high risk project. Given the firm’s potential choices, the bank has two possibilities: screening or not screening.

We now have to determine the equilibrium, that is the optimal strategies of the bank and the firm, depending on the value of parameters. The bank is indifferent to screening or not screening if:

$$\pi_b^s = \pi_b^n$$

$$\iff p_l \gamma(r - r_f + t) - p_l(1 - \gamma)kx_f - s = p_l \gamma(r - r_f + t) + (1 - p_l)\theta(r - r_f + t) - (1 - p_l)(1 - \theta)kx_f - p_l(1 - \gamma)kx_f$$

$$\iff \rho \leq r_f - t + \frac{(1 - \theta)}{\theta}kx_f - \frac{s}{\theta(1 - p_l)}$$

Given equation (8), we are now able to determine two different equilibriums\(^{18}\), that we next characterise:

- When the loan interest rate is greater than the calculated value \((r > \rho)\), that is for high values of interest rates, the return from a successful loan is so high that the best bank strategy consists in not screening the project, \(p_s = 0\). The optimal strategy is therefore for the firm to choose the high risk project \(p_l = 0\). We have a degenerated Nash equilibrium.

\(^{18}\) See appendix B for a graph.
- When the loan interest rate is equal to the calculated value \((r = r\%\)\), the bank does not have a dominant strategy with regard to its choice of screening or not (indeed the profit is the same under both strategies). In such a situation, the firm does not have, neither, a dominant strategy when choosing for the low or high risk project. The probabilities that the bank screens and the firm chooses the low risk project are determined under a pure Nash equilibrium.

- When the loan interest rate is lower than the calculated value \((r < r\%)\), then the bank decides to always screen and \(p_s = 1\). If the bank always screens, then the firm will always choose the low risk project, and \(p_l = 1\). However, if \(p_l = 1\), it is no longer optimal for the bank to screen, as it is costly. And if the bank does not screen, then \(p_l\) is no longer equal to one. In fact, in such a situation, there is no equilibrium.

### 4.2. The degenerated Nash equilibrium

With regard to the banking literature, competition in the credit market is considered as imperfect.\(^{19}\) Therefore we will consider in this section and the next one the case of a monopoly lending market and a monopoly market for banking services.

Our aim will be, among others, to analyse the lending rate, \(r\), and its determinants. In the previous subsection, we have underlined that two equilibriums can occur. The objective of this section and the next one is to determine the value of the lending rate under either equilibriums, as we assume that exogenous constraints will enforce or not the bank to choose the low risk equilibrium (i.e. the pure Nash equilibrium).

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\(^{19}\) The credit market is often considered as monopolistic or oligopolistic in US and Europe (Lewis and Pescetto, 1996, ECB, 2000b).
4.2.1. Screening and project risk

For a value of the credit rate higher than $\frac{\theta}{h}$, the bank does not screen the loan ($p_l = 0$).

The firm profit is $\gamma(l - r)$ if it has chosen the low risk project, or $\theta(h - r)$ if the high risk one is selected. Therefore the firm will always choose to undertake the high risk project (given assumptions on $\theta$, $\gamma$, $h$, $l$). And thus $p_l$ equals zero ($p_l = 0$).

4.2.2. Interest rate setting and risk implication

The entrepreneur has no choice other than borrowing from one specific bank. Therefore the bank chooses the loan interest rate which maximises its profit subject to participation of the firm. The firm’s profit will always be positive or null, given assumptions 1 through 3 of the model.

As seen previously, $r$ is higher than $\frac{\theta}{h}$ and the probability of screening $p_s$ equals zero and the probability of choosing the low risk project $p_l$ equals zero.

The bank profit can be written as follows:

$$\pi^n = \pi_m^n = \theta(r - r_f + t)(1 - \theta)kxf$$

Profit is an increasing linear function of the lending rate $r$. Therefore the interest rate that will maximise the bank profit under the firm’s participation is the one that will seize all the firm’s profit.

**Proposition 1.** A first equilibrium is no screening from the bank and only the high risk investment is undertaken by the firm. The bank’s profit is then maximum for $\frac{\theta}{h} = h$. 

Incentives to screen and the lending rate at equilibrium are not affected by the sale of services.\textsuperscript{20}

In such equilibrium, the bank will capture the entire firm’s profit. The interest rate, charged here, is the same as when no service is sold, but the bank’s profit is higher. This increase in profit depends on the level of commissions and fees earned from services, and on the probability that the high risk project succeeds.

4.3. The pure Nash equilibrium

4.3.1. Screening and project risk

In this case, the probability that the bank screens the loan is no longer equal to zero. We now have to determine the strategy of the firm given that \( p_s \) is positive.

The firm is indifferent between undertaking the low risk project or the high risk project when:

\[
\pi_l^F = \pi_h^F
\]

\[
\Leftrightarrow \gamma (1-r) = (1-p_s) \theta (h-r)
\]  \hspace{1cm} (9)

The choice of the firm will depend on the bank’s behaviour with regard to screening.

From (9) we get the probability of screening, \( p_s^* \), and from (7) we get the probability of choosing the low risk investment, \( p_l^* \). We obtain therefore at the equilibrium:

\textsuperscript{20} The value of the lending rate, as well as the value of the probability of screening (\( p_s = 0 \)) and the probability of choosing the low risk investment (\( p_l = 0 \)) are not affected by the sale of services.
\[ p_s^* = 1 - \frac{\gamma(l-r)}{\theta(h-r)} \]  
(10)

\[ p_l^* = 1 - \frac{s}{(1-\theta)k_{r_f} - \theta(r_{-r_f} + t)} \]  
(11)

4.3.2. Interest rate setting and risk implication

At this equilibrium, the probabilities that screening takes place and that the firm chooses the low risk project are respectively:

\[ p_s^* = 1 - \frac{\gamma(l-r)}{\theta(h-r)} \]  
and  
\[ p_l^* = 1 - \frac{s}{(1-\theta)k_{r_f} - \theta(r_{-r_f} + t)} \]

Given the definition of Nash equilibrium, for these values of \( p_s^* \) and \( p_l^* \), the bank is indifferent between screening and not screening, that is the level of profit is the same in both cases. Therefore we choose the simplest way of writing profit in order to determine the level of the lending rate:

\[ \pi^b = p_l^* \gamma(r_{-r_f} + t) - p_l^* (1-\gamma)k_{r_f} - s \]

In order to assure a positive bank profit we made the following assumption (see appendix C):

Assumption 4

\[ \left[ \sqrt{(1-\theta)k_{r_f} - \sqrt{s}} \right]^3 \geq \frac{\theta}{\gamma} s \]

Proposition 2. The value of the lending rate which maximises the bank’s profit is:

\[ r = r_f - t + \frac{1-\theta}{\theta} k_{r_f} - \frac{1}{\theta} \sqrt{s k_{r_f} \left(1-\frac{\theta}{\gamma}\right)} \]

The lending rate depends on a new variable, which is the commissions and fees charged by the bank while selling services. More precisely, the credit rate is a
decreasing function of the sale of services, that is a decreasing function of the net profit that the bank makes on services. As the disutility of changing the supplier of services, $t$, increases, the bank can set up a lower interest rate on the loan market.

At this equilibrium, when implementing the new value of the interest rate found inside the probabilities of choosing the low risk project and screening found in subsection 4.3.1, we find that the probability that the firm chooses the low risk project remains identical to the situation where the bank sells only loans; but the probability that the bank screens the firm’s project decreases.\textsuperscript{21} This distortion in the bank’s incentives to screen increases, that is the bank has less incentives to screen, as the agent’s transport cost and/or switching cost raise.

Thus, the lending rate is less than the one charged when the bank does not sell services, and the difference comes from the existence of services. Because of the revenue the bank gets from its service activity, it can charge a lower interest rate on loans. Therefore, we can say that services subsidise the credit activity of the bank. In other words, loans are used as loss leader to attract clients.

To summarise, in the case of a monopoly lending market, the sale of services does impact on the lending rate charged in the pure Nash equilibrium. Therefore we observe that the sale of services may induce, as a strategy for banks, a decrease in their lending rate in order to attract new clients. We also observe a change in the risk incentives of banks: by decreasing their probability of screening, banks are willing to take on more risk.

\textsuperscript{21} When $t$ increases, the lending rate decreases. And the probability that the bank screens is an increasing function of $r$. Thus when $t$ increases, $p_s$ decreases.
5. Conclusion

This paper investigates the effect of banks’ service provision on the behaviour of banks and firms when considering lending relationships. Our theoretical model implies that banks which sell services, cross-subsidise their lending rate. In other words, banks strategically adopt the use of loans as loss leaders in order to gain new borrowers, and therefore new clients willing to pay fees. Further, banks’ incentives to screen decrease when banks sell services, while firms’ behaviour is not modified. As a consequence, the desire to sell services leads banks to take on more default risk. Hence a strategy aiming at further increasing the sale of services implies higher credit risk. The results reported in this paper suggest that regulators should take account of service activities when monitoring credit risk. Risk related capital adequacy requirement may well need to be tightened as a consequence.
Appendix A. Game tree

Monopoly Lending

The bank is price-maker.

Lending rate: \( r \)

---

Firm expected earning

\[
\begin{align*}
\text{low risk project: } & (l-r) \times \gamma + 0 \times (1-\gamma) \\
\text{high risk project: } & (h-r) \times \theta + 0 \times (1-\theta)
\end{align*}
\]

The bank does not lend funds to the firm.

Total bank expected earning

\[
\begin{align*}
\text{screening: } & (r - r_f + t) \times \gamma - k_r \times (1-\gamma) - s \\
\text{not screening: } & (r - r_f + t) \times \gamma - k_r \times (1-\gamma) - s
\end{align*}
\]

\[
\begin{align*}
\text{screening: } & (r - r_f + t) \times \theta - k_r \times (1-\theta) \\
\text{not screening: } & (r - r_f + t) \times \theta - k_r \times (1-\theta)
\end{align*}
\]
Appendix B. Agents’ best reaction functions and equilibriums.

probability that the firm chooses the low risk project, $p_l$

best reaction function - firm

no equilibrium

best reaction function - bank

pure Nash equilibrium

degenerated Nash equilibrium

probability that the bank screens the demand for loans, $p_s$

Appendix C. Assumption 4.

The bank’s profit is positive when:

$$\pi^b = p_l \gamma (r - r_f + t) - s > 0$$

To obtain assumption 4, we need to replace $p_l$ by its value and to simplify the expression.
References


