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Long Distance Surface Wave Propagation Over Realistic Terrains

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Abstract—Full-wave time domain simulation on very large terrains and complex media can help to solve communication channels or radar detection problem on wide frequency range but can cost very large resources. Specific terrain data is needed to generate the scene model which can also be computationally expensive. Depending on the frequency of interest, some terrain model such as forests may be simplified using equivalent models to reduce the resource cost. Finite Difference Time Domain scheme is used with the Moving Window technique to limit the computational domain following the wave front. The computation cost is significantly lower than the solving Maxwell equations over the whole domain. These three main points are developed in this paper, presenting challenges and solutions of the project which aims to build a software capable of time domain simulation of a surface wave over long distance through complex media.

Index Terms—FDTD, modeling, simulation

I. INTRODUCTION

Surface wave propagation on realistic terrain involves numerous complex media such as vegetation, humidity, ground, water area, but also diverse geometries from ground surface to trees, as shown in Fig. 1. These media introduce many wave scattering, diffraction and dispersion effects, among others, complicating wave signal analysis such as telecommunication problems.



Fig. 1. Complex environment graphic for ground telecommunications.

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To gain a better control and insight for such cases, rigorous methods are necessary to model relevant environment elements over a large area. For example, Sommerfeld integral method is able to deal with diverse surface wave propagation on different ground surfaces but is limited to homogeneous or stratified grounds with a planar surface overground.

This project aims to build all the necessary tools packaged in a simulation software to be able to efficiently simulate fullwave over large distance such as several thousands of wavelengths. Realistic terrain will be modeled in 3D, with necessary data precision level to cover mainly land communications in the HF, VHF and UHF frequency ranges. Emitter, receptor and their environment are also included in the simulation. Since mainly rural areas will concern this project, forest modeling for electromagnetic wave simulation is an essential part of the scene generation but raises concerns about large vegetation data volume storage and processing. On a certain frequency range, homogenization techniques to obtain equivalent forest models [1] will considerably reduce these resources costs. The Finite Difference Time Domain (FDTD) solver will be adapted for very large domain simulation using the Moving Window technique [2] thus called MW-FDTD. With this technique, electromagnetic fields are computed in a limited domain following the wavefront. The overall computation cost is significantly lower than the full domain simulation but at the expense of wave truncation that may result in inaccuracies on field prediction.

Specifically in our context on surface wave propagation, this technique is used by the authors of [3]–[5] exhibiting concerns about propagation with grazing incidence on absorbing boundaries, through forests or on ground surface roughness. Numerical dispersion error will substantially increase for very long domain simulation [6] where phase correction is shown to be sufficient but limited in application. Higher-order FDTD schemes [7]–[9] mitigates such errors while also being more accurate on coarser meshes.

This paper is structured on three core parts of the software : generation of the scene model, from terrain data processing to

simulation scene model, with a focus on vegetation; the electromagnetic model centered on vegetation model which must be simplified for valid frequency ranges; the moving window technique for the FDTD solver with domain decomposition.

II. SCENE MODEL GENERATION : FROM TERRAIN DATA TO MODEL

Structured mesh of realistic scenes model built from real terrain data are required for this project. Terrain data can be usually obtained from a geographic information system (GIS), with various data types for different level of representation and information. Ground and above-ground data are contained in separate model : digital terrain model (DTM) and digital surface model (DSM). These models come in various formats such as vector data, raster data or point clouds identified as categorical data (vegetation, structures, basins, etc...). GIS operates on such large data volume which can be heavy on resources for data storage and processing. As we focus on rural zones, a large portion of such data is forest data. This data volume depends on the data acquisition method : for example, airborne or ground LiDAR differs as the former can have difficulties to acquire data located in the forest (e.g. classified data as low and medium vegetation); and the latter yield more data between ground and forest canopy. Numerical resource consumption is reduced if we restrict vegetation data to only relevant data for the electromagnetic model.

Thus, simplifying forests is a must and need to be correctly constructed to obtain a valid electromagnetic model. Two models are identified for this project : homogeneous or discrete model. The homogeneous model is the simplified model of the forest and is defined as dielectric slabs inside the forest envelope with effective dielectric parameters. Here, we will focus on the discrete model as the next section will detail on deriving such models.

The discrete model is the realistic model of the forest, with branches or even leaves. For this project, discrete models are reserved for construction and validation of equivalent forest models. As the original GIS data do not provide a full tree model, tree reconstruction or generation techniques [10] are used to build tree skeleton and then tree model. Among such techniques, two simple techniques were investigated : the space colonization [11], [12] and the Weber-Penn algorithm [13]. The first one connects a point cloud in a way resembling to tree branching. The second one generates a tree from a list of parameters and rules, which is similar to L-systems [14]. The space colonization requires more data points to build realistic trees. The Weber-Penn algorithm do not directly use data points, as the only requirement are a stem position and parametric data which effect on tree appearance is less straightforward. A representation of tree skeletons using space colonization on a LiDAR HD point cloud of France National Institute of Geographic and Forest Information (IGN), is shown in Fig. 2. In the same figure, an illustration of a 1 cm structured cubic mesh of trees generated with a Weber-Penn type algorithm [15] is displayed.

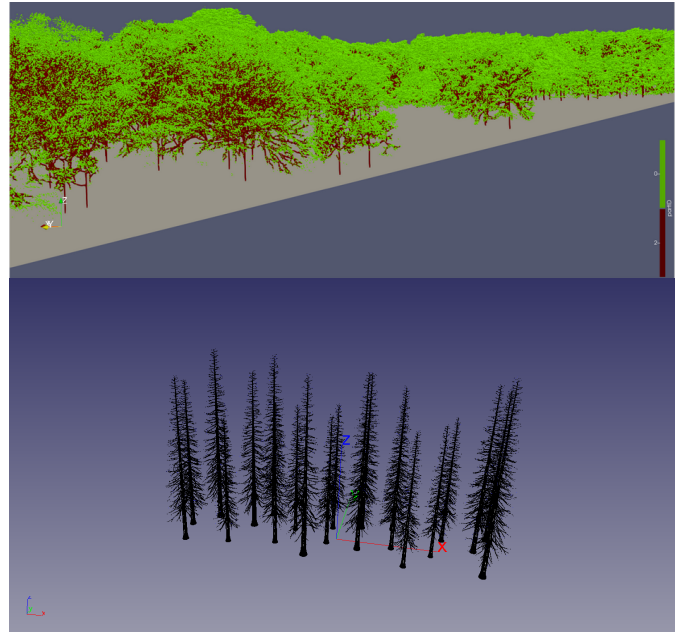


Fig. 2. Top : Graphic representation of tree skeletons from airborne LiDAR point cloud (France IGN LiDAR HD) using space colonization : segments are transformed to cylinders, and leaves are represented by random spheres. Bottom : structured mesh of trees generated with a Weber-Penn type algorithm.

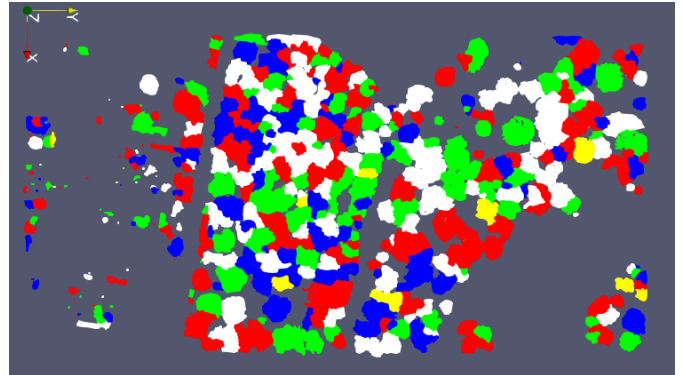


Fig. 3. Tree segmentation from airborne LiDAR point cloud (France IGN LiDAR HD).

Currently, the current process to build the scene model is to extract DTM and DSM data, to create for each a surface mesh with tessellation then to build structured meshes, structured volume meshes and finally forest generation data. To build forest data, the usual process [16] is to detect individual trees, by extracting each tree trunks location inside forests using a maximum filtering. Tree crowns are delineated with a tree segmentation algorithm (watershed algorithm). Finally, trees are built or reconstructed from tree stems inside each tree crown. An illustration of such tree segmentation using Digital Forestry Toolbox [17] is shown in Fig. 3.

III. SIMPLIFIED FOREST MODEL : HOMOGENEOUS MEDIA

The modeling of propagation of electromagnetic waves inside complex environments such as forest is numerical

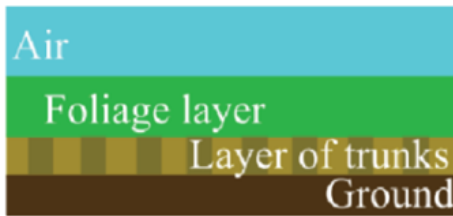


Fig. 4. Tamir vegetation model

resource consuming when a realistic scene has to be modeled. In our case, long distance propagation over ground is prone to encounter forests. Two main reasons make such a modeling intractable:

- Huge data volume is required to store data on forest (trees, branches and leaves characteristics : geometry, composition ...) over a large zone,
- the numerical modeling would need a very fine discretization of a large volume, thus inflating required computational resources.

To overcome these problems, it is necessary to define equivalent models of the additional wave propagation delay and attenuation due to the forest : the simplest models replace the envelope of the original media with a homogeneous dielectric slab. The geometry simplification greatly decreases the data volume used to model the forest.

As the frequency range is large, from HF to UHF frequencies, different models has to be defined. In this paper, we will introduce the two main constituents of the equivalent models:

- the geometric shapes of the new dielectric bodies,
- the electrical parameters (effective permittivity and conductivity) of each dielectric body.

Considering a forest, one of the well-known ways to give an equivalent representation is to use the model introduced by Tamir [1]. In such representation, the forest is modeled with equivalent lossy dielectric slabs as shown in Fig. 4.

Usually, two kinds of description are defined depending on the frequency:

- The three slabs model: ground, forest, air. This is valid in the low frequency range and assumes no depolarization due to tree trunks.
- The four slabs model: ground, trunk, canopy, air. Here, the effects on wave propagation of the trunk (which couples more for the vertical polarization incident field) and the canopy (scalar homogeneous model valid for the three-axis direction) are individually modeled as distinct dielectric bodies.

Concerning the electrical aspect, different models have been developed to realize the homogenization. This one concern the forest slab for the first model and the canopy slab for the second model. The most famous homogenization approach is the Maxwell-Garnett formula [18]. Such a homogenization is generally used for only one kind of inclusion (biphase slab) in a host medium. It also assumes that the density of inclusions is very low, typically lower than ten percent of the

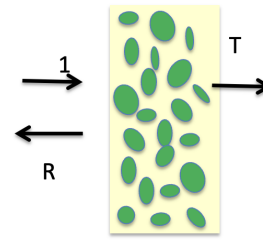


Fig. 5. FDTD modeling of a slab containing inclusions

host volume. This can be true when dealing with branches surrounded by air in a canopy. More similar formula such as the Bruggeman formula can deal with more general problems. This can be used for multiphase media, where the host and the different kinds of inclusions are represented in the same way. In [19], such models are generalized by the definition of a parametric formula. It is interesting to see that the Maxwell-Garnett and Bruggeman formula are included in this model by taking particular values of a parameter.

An approach [20] based on the Foldy approach consists in taking into account the diffraction of an aleatory infinite distribution of branches and leaves around a given virtual sphere in the canopy, where an incident plane wave is considered. From the mean field obtained inside this sphere the complex constant propagation can be derived.

In our numerical experiments, we have seen that the analytical approaches such as Maxwell-Garnett or Bruggeman ones were not adapted to our problem as they give inaccurate results. Then the Foldy approach was experimented. A model developed by Sihvola [19] shows that the problem can be solved analytically, provided that the analytical form of the diffraction coefficient is known for branches and leaves. The authors choose to model branches as long cylinders and leaves as flat cylinders. Knowing their density in the host medium and considering their aleatory distribution and orientation defined by a probability density generally inferred from experimental observations, the mean field inside the canopy can be calculated. Supposing this field is the same everywhere (infinite, homogeneous material), then the complex propagation constant can be derived and thus the associated complex permittivity. The main problem of this method is to quantify the electrical characteristics of the constituents.

In [21], a similar model of randomly distributed inclusions in a dielectric slab is presented with the same objective to derive the effective complex permittivity of the equivalent homogeneous dielectric slab. The inhomogeneous slab is submitted to a plane wave and the reflection and the transmission coefficients are then solved numerically, as shown in Fig. 5. This approach is thus more general than the previous one and can give access to statistical distributions of branches and leaves characteristics derived from experimental Lidar data measurements. For the homogeneous slab, the reflection and transmission coefficient are known analytically. The effective complex permittivity is then derived by matching the analytical

and numerical, reflection (or in our case transmission) coefficient. As shown in [19], if a mixed media can be modeled with the Maxwell Garnett formula, its frequency variation can be modeled by a Debye filter. In the present case, this formula is not always accurate. Then attempts to fit frequency variation of the complex permittivity with the vector Fitting approach led us to conclude that a couple of two first order filters is required.

As a filter model need extra variables in the FDTD scheme, another strategy has been followed. Looking closely to the complex permittivity computed from FDTD simulations, it appears that its imaginary part behaves as a capacitance. From this observation, a model including only one Debye filter and a conductivity term was considered [22], [23]. The different parameters of the following formula have been optimized to minimize an objective function which is the sum of the distance between the numerical and analytical reflection coefficients and the similar distance between transmission coefficients.

$$\epsilon_r = \epsilon_\infty + \frac{\epsilon_s - \epsilon_\infty}{1 + j\omega\tau} - j \frac{\sigma}{\omega\epsilon_0} \quad (1)$$

The Fig. 5 and 6 give the values of the complex permittivity optimized after FDTD simulation and the one fitted with the previous procedure. In the present case the characteristics of the inclusions are : $\epsilon_r = 3$ and $\sigma = 10^{-3}$; the volume of inclusions sums to twenty percent of the total volume and the slab length is 6.5 m.

IV. MOVING WINDOW – FDTD

Computing the EM field of surface wave propagation over a large zone is inefficient especially during the initial period of the wavefront traveling, where most of the computational domain has zero field. Domain decomposition techniques with field teleportation can be used to solve this problem as in [24]. A somewhat more straightforward solution is the moving window technique [25], [26], which restricts the computational volume to the wavefront. This volume follows the wavefront movement by updating medium values inside the computational volume. In both cases, unneeded field computation are reduced. Memory consumption is kept low since computational data are only loaded in the smaller computation volume and updated during propagation. The MW-FDTD has been successfully operated for surface wave propagation through forest models [27] or urban areas [8].

Usually, the initial window size is set before calculation and kept through the whole simulation. If the window size is not large enough, wave truncation can happen during wave propagation depending on terrain conditions. In such cases, significant inaccuracies in field computation at the target position may occur. Larger window size reduces such inaccuracies at the cost of increased computation resources [28].

Implementation of the moving window technique can be straightforward since it involves mainly updating and shifting field and medium values. Extra care must be taken for shifting values in computer memory as this can become heavy on

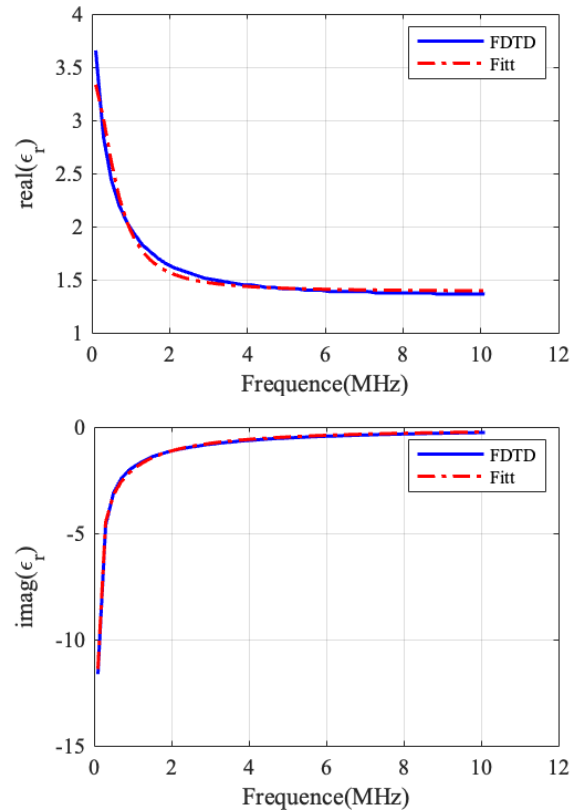


Fig. 6. Real (top) and imaginary (bottom) part of the effective permittivity for randomly (uniform) distributed inclusions of dielectrics ($\epsilon_r = 3$ and $\sigma = 10^{-3}$) with fractional volume of 20%.

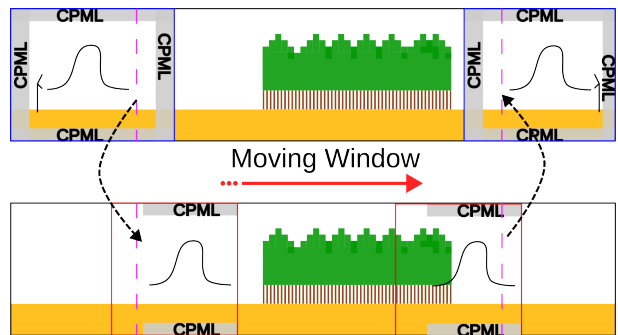


Fig. 7. Domain decomposition in three parts : Emitter and Target zones (blue frames) and moving window (red frame).

computation if done at almost every iteration. The current implementation uses an additional buffer extending fields and medium arrays which allows shifting arrays only when this buffer is completely used. One can choose to implement the moving window technique differently : in [29], this technique is used in the segmented FDTD scheme. The segmentation comes from splitting the computational domain in two distinct segments, with absorbing boundaries. The segments are initially joined at the start of the wave propagation from the first segment to the second one. When the wave arrives at the end of the second segment, the first segment is placed at

the end of the second one. The wave propagation continues following this recursion.

Here, the MW-FDTD is currently implemented with a certain domain decomposition as shown in Fig. 7. The decomposition splits the domain in three zones enclosing : the emitter, the target, and the full domain without the emitter and the target where the moving window will follow the surface wave propagation. A Huygens surface is used to transfer fields between domains. Only forward propagation in the recipient domain is generated from the transfer surface thanks to Huygens properties. This splitting allows us to use different FDTD solvers in each domain: standard FDTD solver in the emitter and target domains, the moving window FDTD solver (possibly higher order) in the full domain. Solvers run concurrently only while their field transfer is active.

Notice the two boundaries conditions at the front and back of the moving window in the propagation direction : no boundary conditions are applied during the window movement. At the back boundary, as noticed in [25], field computation may be erroneous but does not propagate in the interior of the window since the window truncates field values at its back. At the front boundary, field must always be null, thus does not need any specific boundary conditions.

Horizontal wave propagation induces highly grazing incidence for absorbing boundary conditions [24], [27]. At such incidences, wave absorption may prove to be difficult but must be absorbed enough for numerical noise to not cover the main signal. For short distances, optimized profile of CPML boundary conditions for grazing incidence may prove to be sufficient as we successfully tested propagation over hundreds of wavelengths. For very large distances, PML performance is too low [27], even with optimization since incidence angles becomes lower and lower as the distance grows [24]. Thus, specific absorbing boundary conditions must be derived such as the reradiating boundary condition [24].

Very long distance wave propagation simulation is prone to numerical dispersion error [6]. High order FDTD scheme used with the window technique significantly mitigates numerical dispersion with better results as shown in [7], [8]. Comparable results should be obtainable with the high order FDTD scheme in [9] as it also exhibits low numerical dispersion error. Moreover, its numerical dispersion error is optimizable for a chosen frequency range and propagation direction. This is valuable in this context of a mainly mono-directional surface wave propagation.

V. NUMERICAL EFFICIENCY

Results presented here are preliminary results obtained from current development. A simple surface wave propagation through some dielectrics slabs over ground is computed with the standard FDTD and FDTD-MW scheme. The volume is 10500x3x300 cubic cells with cell side length of 10 cm. Gaussian excitation is placed at 25 m in the x direction, for x forward wave propagation. CPML layers are placed on all boundaries and at least 21K iterations are computed. Simulations run with eight CPU cores, for various window size.

TABLE I
SIMULATION STATISTICS FOR 10500X3X300 CELL VOLUME

Window Size (cells)	FDTD	FDTD-MW		
	N.A.	1000	2000	3000
Average time (1K iters)	6min31s	1min13s	1min53s	2min15s
Memory (GB)	6.25	1.38	2.19	2.94

Average computation runtime for 1K iterations are reported in table I alongside program memory usage.

FDTD-MW is largely faster than the full domain computation, where acceleration can scale up to x5. Memory usage is also significantly lowered, and can be adjusted to fit limited-capacity memory computers. Currently, the trade-off is the field computation accuracy as the window may truncate a part the signal.

VI. CONCLUSION

An overview of the project on long distance surface wave propagation was presented. Three main points were covered featuring large data volume processing for scene generation, forest electromagnetic modeling, and the MW-FDTD scheme. Since rural areas are our typical scenes, vegetation data is identified as a potential for lowering numerical resource consumption. Specifically, a simpler model of forests made of dielectric homogeneous slabs was presented. A focus on homogenization techniques validated the derivation of effective electromagnetic parameters for such models. A simple example confirmed that the equivalent homogeneous slab can be modeled with a Debye filter and a conductivity term. On the solver part, the FDTD scheme is used with the moving window technique to significantly lower numerical resource consumption. Related concerns were presented such as absorbing boundary conditions for highly grazing incident waves, and numerical dispersion error. These are topics of very near future development, which also includes extrapolation techniques to improve results accuracy from a signal truncated by the moving window.

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